

The Spectre Knot

A single-strand Celtic knot on an aperiodic monotile

One endless ribbon through a tiling that never repeats

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A tile that never repeats. Squares and hexagons tile the plane periodically: slide the pattern and it lands on itself. Penrose found pairs of tiles that can only tile non-periodically; whether one shape could force this on its own stayed open for decades, as the *einstein* problem (‘one stone’). In 2023 David Smith, Joseph Samuel Myers, Craig S. Kaplan and Chaim Goodman-Strauss solved it, first with the *hat*, which needs some copies flipped over, then weeks later with the **spectre**, which needs none. The tile used here is the straight-edged form of the spectre, their fourteen-sided polygon $\text{Tile}_{(1,1)}$. Laid without flipping, its copies tile exactly as spectres do, and only in patterns that never repeat. Curving its edges gives the strict spectre, which rules out flipped copies by its shape alone. The knot is the same on both.

A knot that never repeats. A Celtic knot is one ribbon woven over and under itself, usually built by repeating a single cell. Here the cell is the spectre. Every tile carries the same small drawing: fourteen *pieces* of ribbon, each from one edge of the tile to another, which join up where tiles meet (page 2). The question is whether one drawing can make every piece on every tile of an infinite tiling join into a **single strand**. On any repeating tiling that is impossible. On the spectre it can be done. In the spectre tiling drawn here, the *nest tiling* (foot of the page), grown outwards from one tile with each block in the same place (child number 1) inside the next, the Spectre Knot gives one endless strand through every tile, with no closed loops and no second strand.

1 Why it is one strand

Spectres group into supertiles of eight or nine, these into larger ones, and so on for ever. Seen from outside, every supertile looks the same: the ribbon hooks round each corner of its outline, except at the two ends of one edge, where two ribbons cross. Where supertiles meet along a run of edges the ribbons zigzag along it, so a run two edges longer changes nothing. The pattern passes up the levels like falling dominoes, no ring can form, and pieces carried up a nested chain of supertiles all end on one strand.

The spectre paper proves every level is built by the same rule (its Theorem 5.1), and the edge lengths at any level are those of the smallest supertiles made longer by even amounts. So one exact computation at the smallest level covers every level. The rest was computed directly too, up to 133 million tiles.

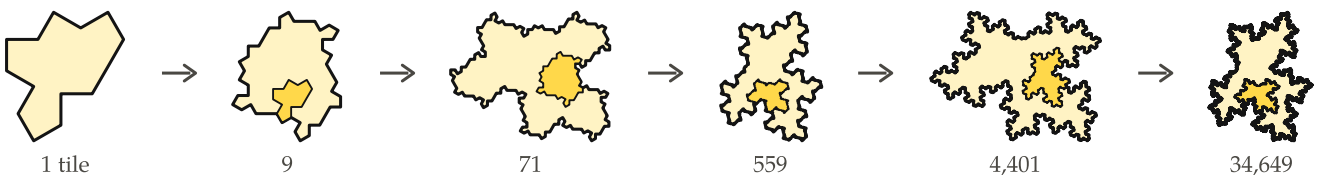
2 Watching it join up

The number of strands that a nested block’s ribbon pieces lie on, as the block around it grows:

block	pieces	1 up	2 up	3 up	4 up
1 tile	14	5	2	1	1
9 tiles	46	10	2	2	1
71 tiles	182	37	2	1	1
559 tiles	758	146	2	2	1
4,401 tiles	3,198	613	2	1	1

The large first numbers are pieces on the stretch of a block’s edge shared with the next block; they join a level later. Odd-level blocks need four levels, not three: one strand ends on an edge shared by the next two blocks up, and waits for a block that encloses it (pages 5 to 7).

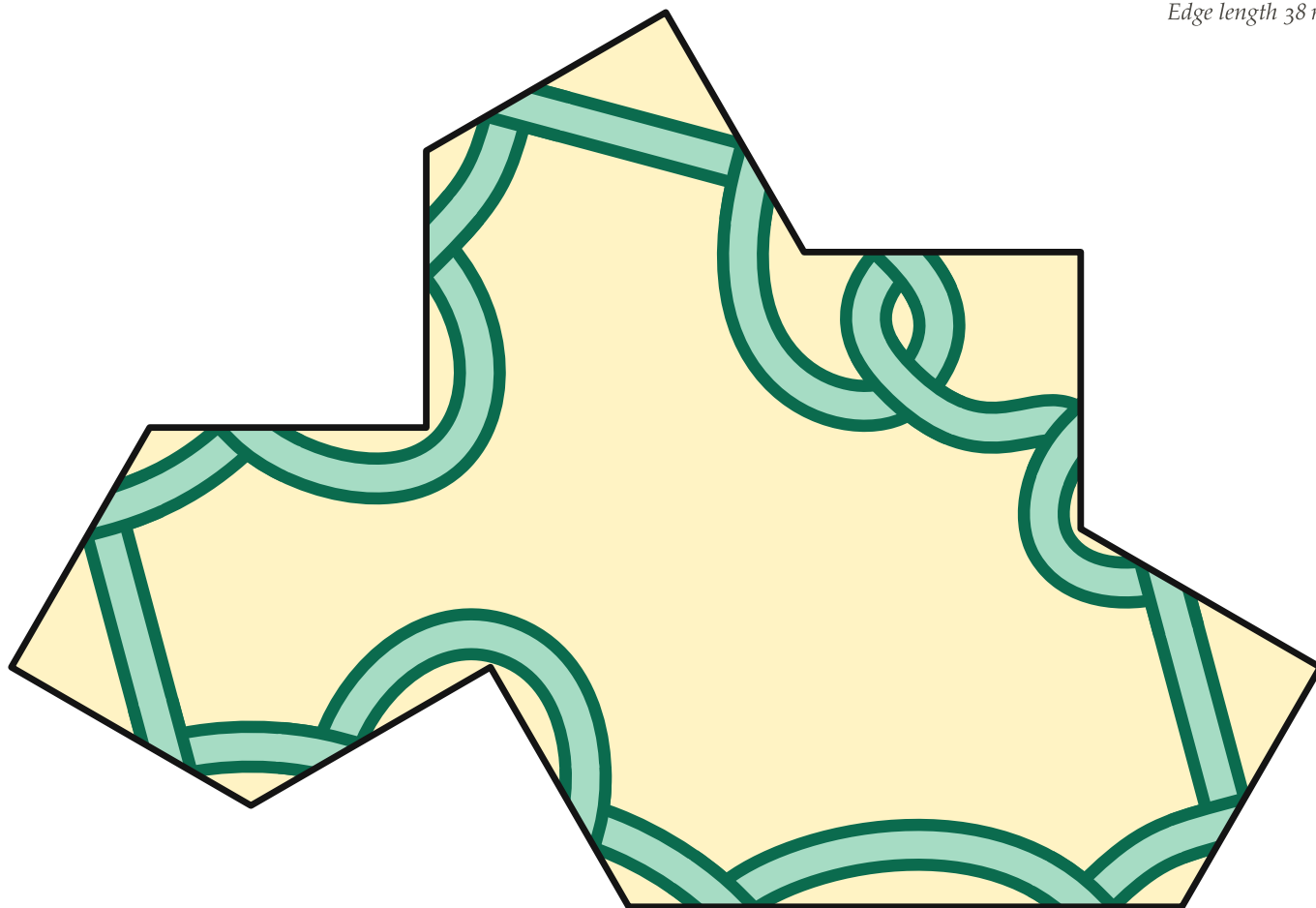
Made with AI, not peer reviewed; how, and what is checked: cortado-j.github.io/SPECTRE_KNOT/made.html. D. Smith, J. S. Myers, C. S. Kaplan, C. Goodman-Strauss, *A chiral aperiodic monotile*, Combinatorial Theory 4(2) (2024), arXiv:2305.17743; and *An aperiodic monotile* (the hat), Combinatorial Theory 4(1) (2024), arXiv:2303.10798. Knot design 0.1:01.



Each block is the shaded part of the next, all the same way up. The chain fills the plane: it is the nest tiling drawn here.

The Spectre Knot • one tile

Edge length 38 mm



One crossing inside

Two ribbons cross at every edge of the tile, at a point shared with the neighbour across it, and they cross just once inside the tile. Fourteen pieces of ribbon join the edges up: twelve short hooks that turn round a corner, and two longer pieces that leave the same edge and cross in the middle.

Off-centre on purpose

Each crossing sits 0.15 of an edge off the midpoint, alternately one way and the other. Spectres always meet even-numbered edge to odd-numbered edge, so neighbours' shifts coincide. It is only for looks: midpoint crossings join the ribbons the same way.

Which tiling?

The spectre tiles the plane in infinitely many ways. One strand (the *Ariadne Theorem*) is proved only for the one drawn here, grown from the nest of page 1. Others whose supertiles fill the plane have no closed loops (the *Daedalus Theorem*), but some seem to have three strands. Tilings with a fault line are not covered.

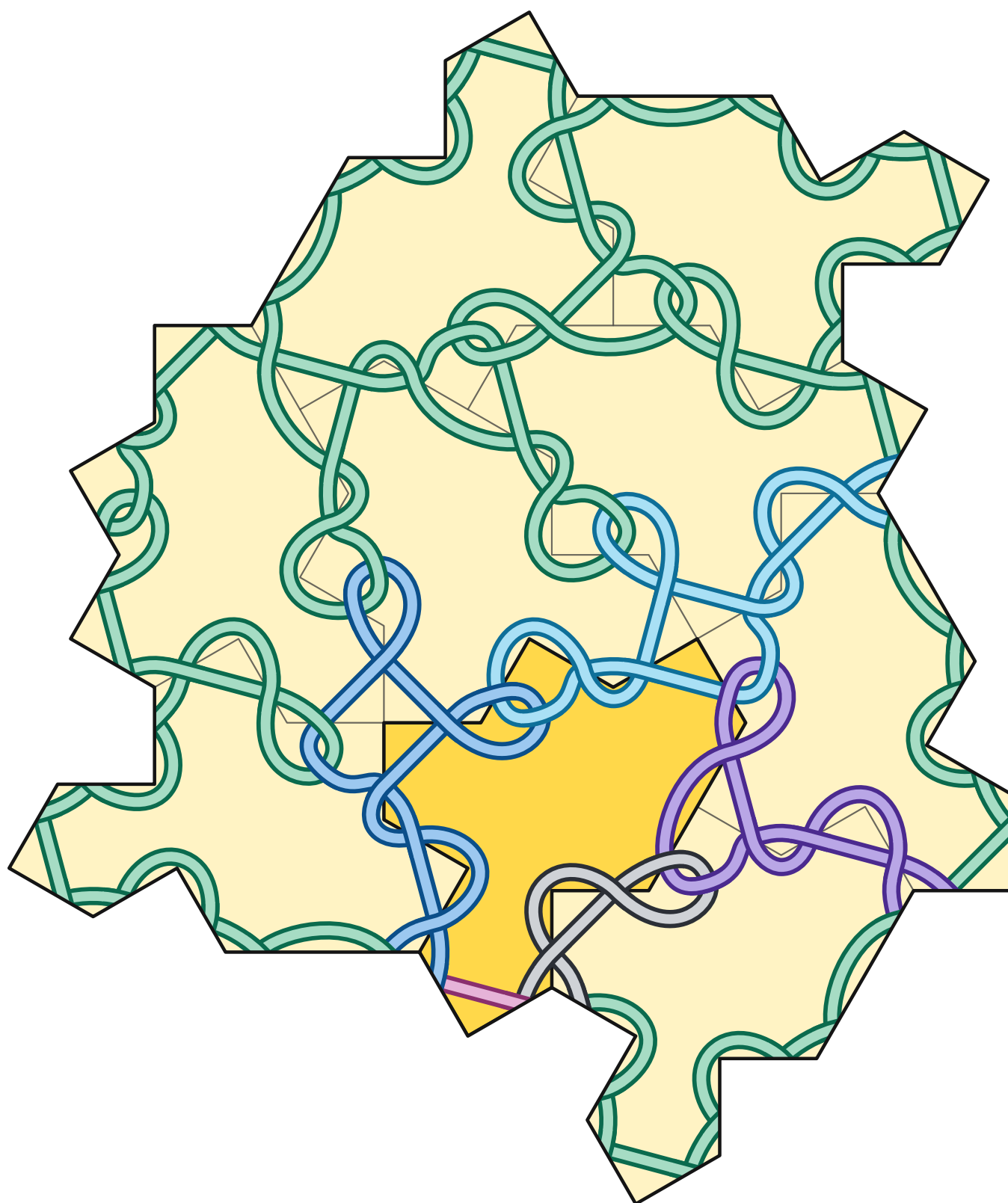
One tile, one strand

Copy this tile across the nest tiling of page 1 and the pieces join into a single strand that visits every tile, never closes up and never repeats. The over-and-under alternates all the way, and one printed tile serves everywhere. The traced tile on the following pages is on one strand by the 559-tile supertile.

Things to try

1. Make a few tiles from page 8 and copy a patch from pages 3 to 7. Follow any ribbon with a finger: it always leaves the patch. Why must it?
2. Show that no design repeated on every tile of a periodic tiling, such as squares or hexagons, can ever give a single strand. Hint: what does a translation do to the strand?

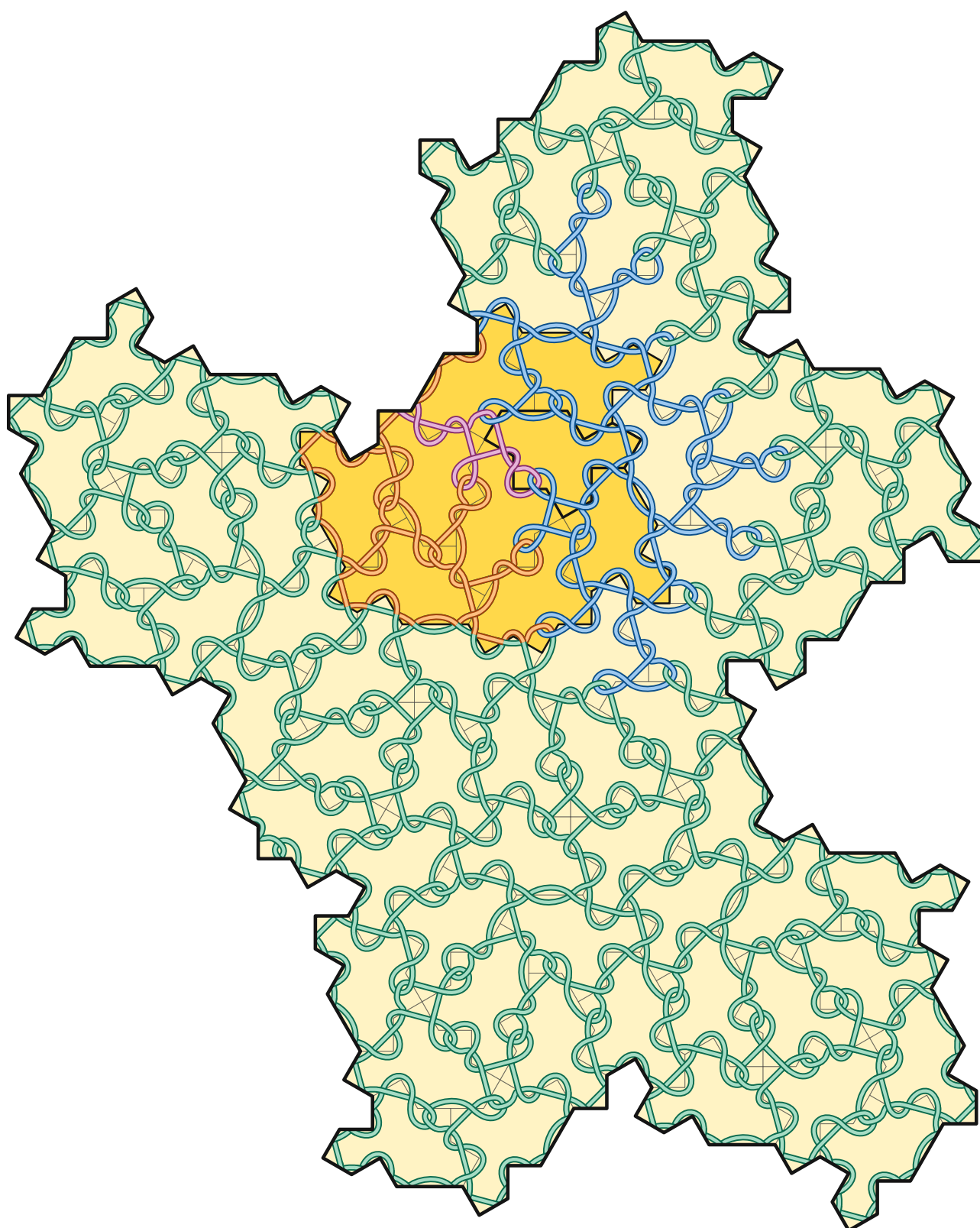
Level 1 supertile · 9 tiles



Outlined in black and shaded brighter yellow: the original tile from page 2. Its 14 ribbon pieces lie on **5 different strands** of this supertile, each drawn in its own colour.

-  a spectre tile
-  the previous page's block
-  the ribbon
-  its ribbon pieces
-  strands through the original tile
-  the original tile

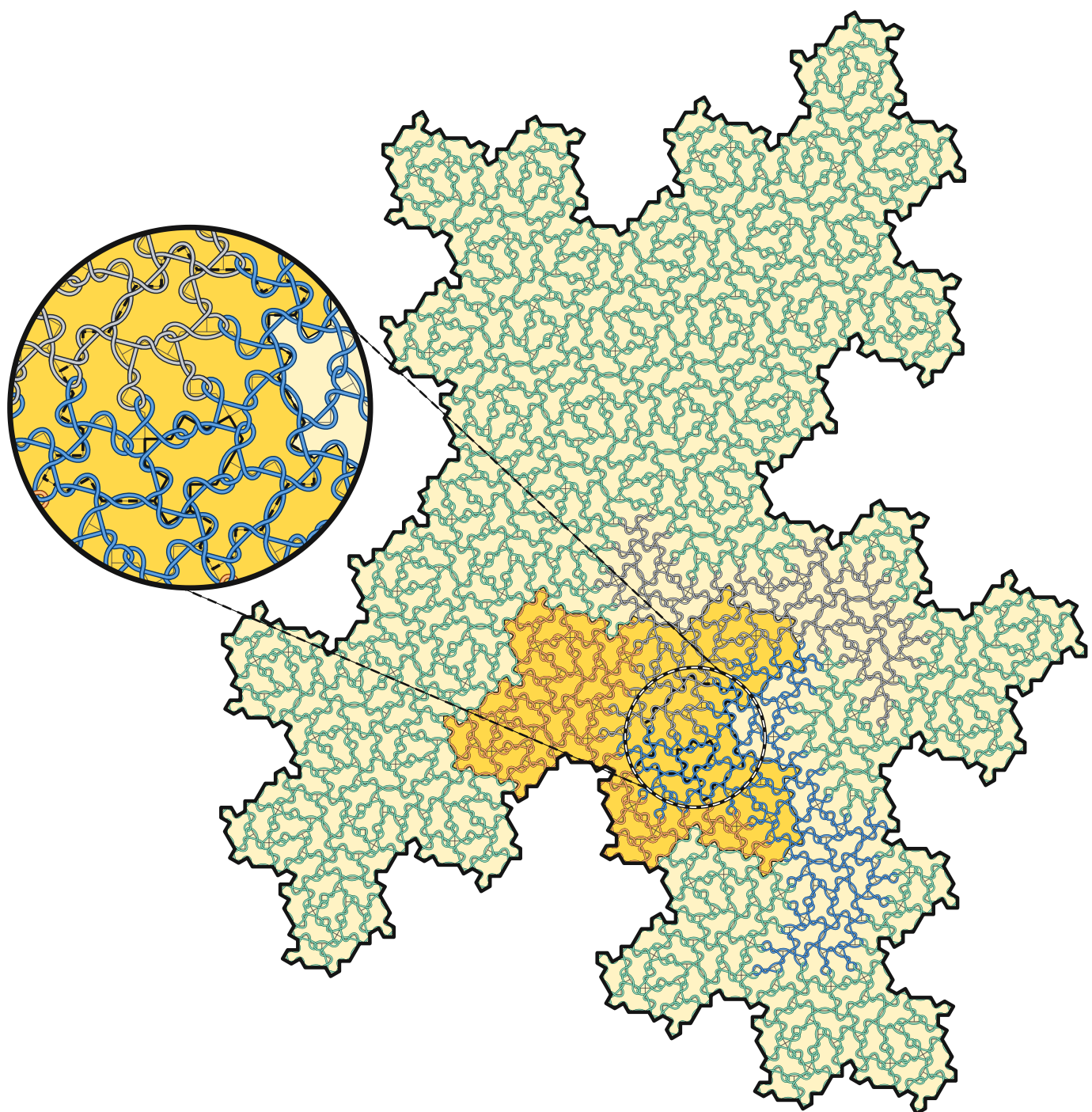
Level 2 supertile · 71 tiles



Brighter yellow: the 9-tile supertile of page 3; its 46 ribbon pieces (orange) lie on 10 strands here. **Outlined in black:** the original tile. Its pieces now lie on just **2 strands**, blue and purple.

- a spectre tile
- the previous page's block
- the ribbon
- its ribbon pieces
- strands through the original tile
- the original tile

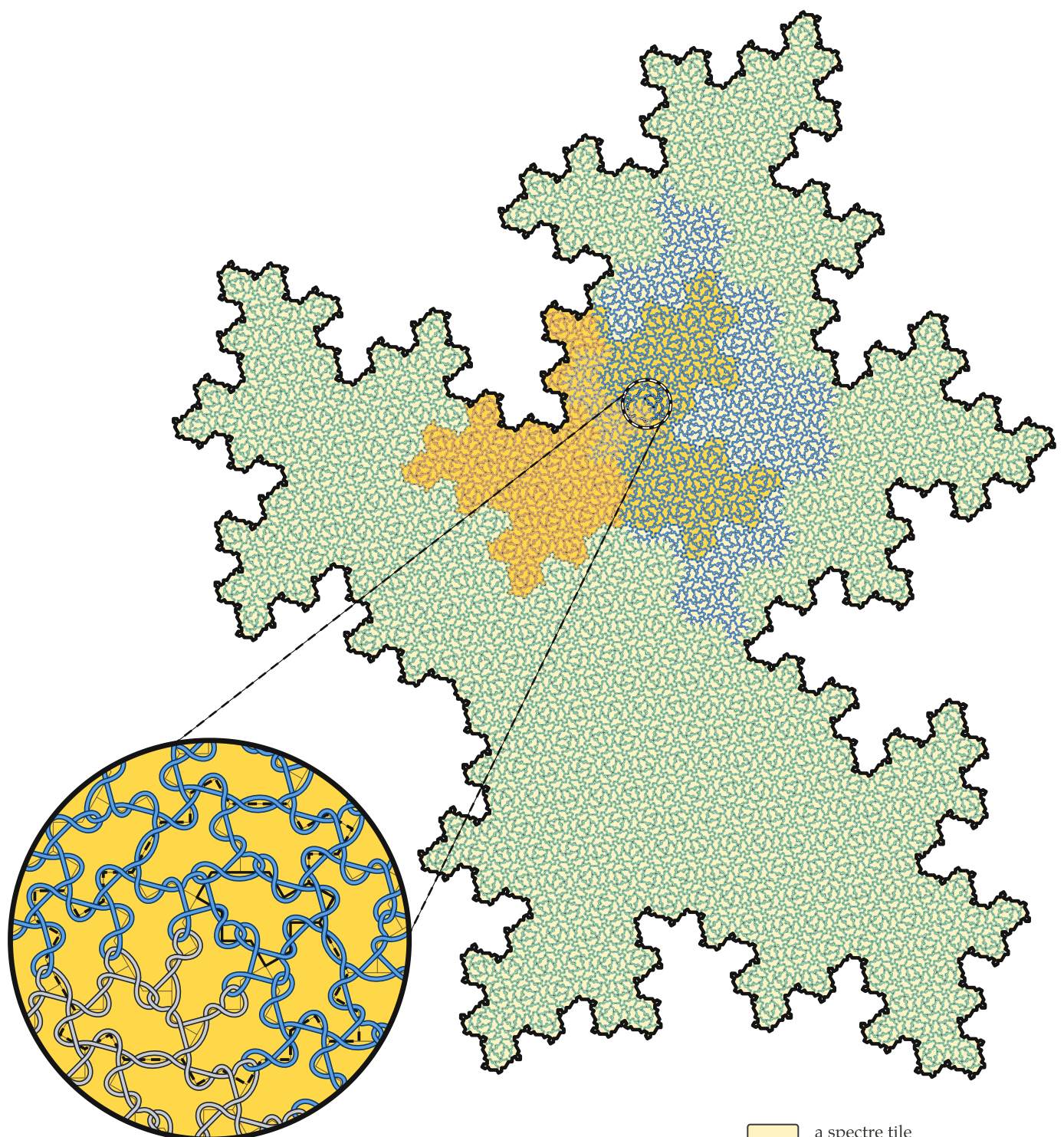
Level 3 supertile · 559 tiles



Brighter yellow: the 71-tile supertile of page 4; its 182 pieces (orange) lie on 37 strands here. **Circled, dashed:** the 9-tile supertile of page 3, with the original tile inside it. The original tile is now on **one strand** (blue, 68 tiles), but the 9-tile block is not: the rest of its pieces lie on a **second strand** (charcoal, 69 tiles). **Inset:** the circle, magnified 3×.

- a spectre tile
- the previous page's block
- the ribbon
- its ribbon pieces
- strands through the original tile
- the 9-tile block's other strand
- the 9-tile supertile
- the original tile

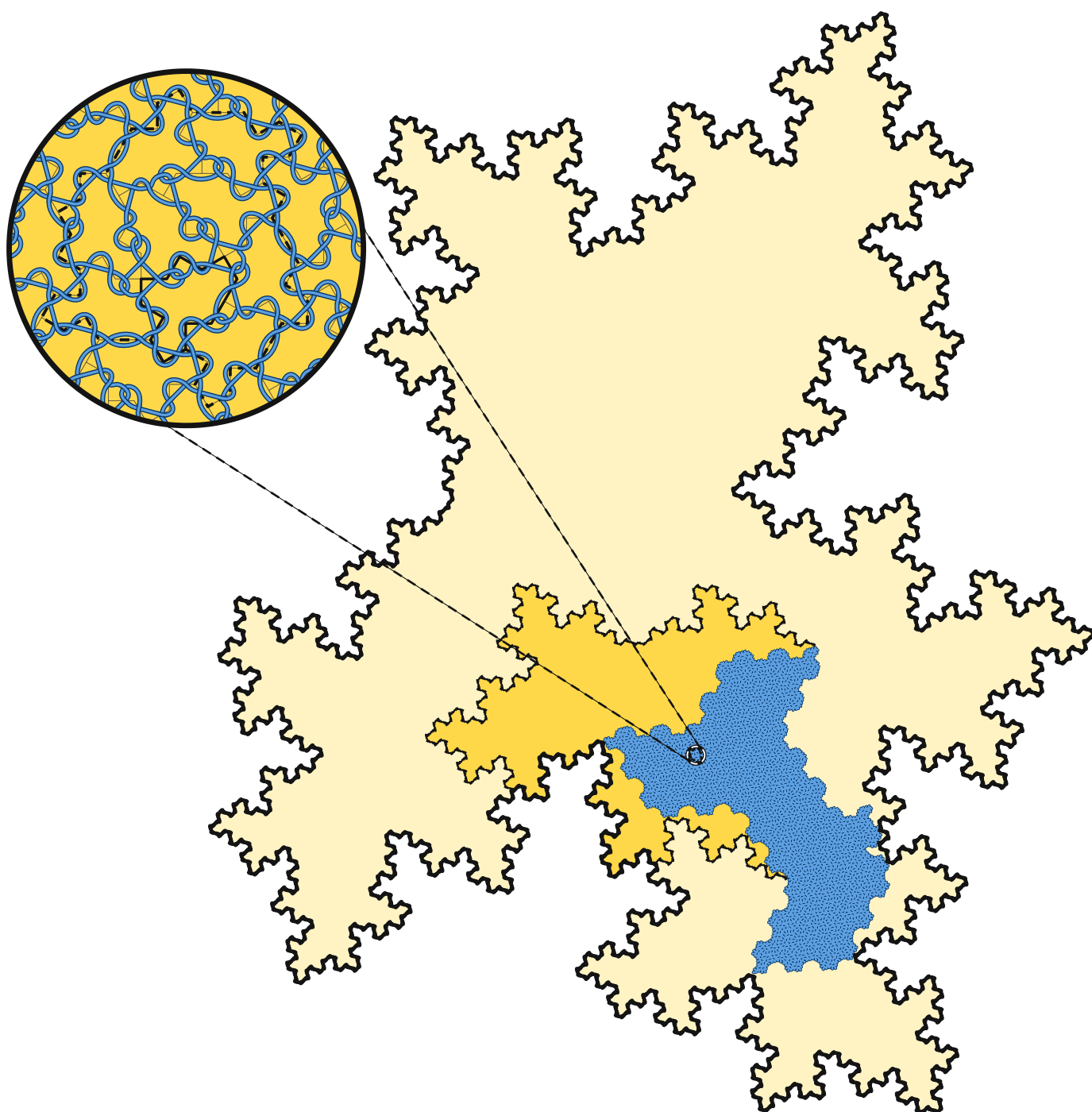
Level 4 supertile · 4,401 tiles



Brighter yellow: the 559-tile supertile of page 5 (758 pieces on 146 strands). The 9-tile block is **still on two strands**. The blue one has grown to 687 tiles; the charcoal one is exactly as before, 69 tiles, because both its ends sit on a stretch of edge that this supertile shares with the 559-tile block. It can only grow once a bigger block encloses that stretch. **Inset:** the circle, magnified 8×.

-  a spectre tile
-  the previous page's block
-  the ribbon
-  its ribbon pieces
-  strands through the original tile
-  the 9-tile block's other strand
-  the 9-tile supertile
-  the original tile

Level 5 supertile · 34,649 tiles



Brighter yellow: the 4,401-tile supertile of page 6. Here at last the two strands meet: all 46 pieces of the 9-tile block, and with them the original tile, lie on **one strand**, which winds through 3,035 tiles (blue). This is four levels above the 9-tile block, which is why the proof climbs four levels: blocks at odd levels need four, blocks at even levels, like the original tile, need three. At this size only that strand is drawn.

Inset: the circle, magnified 20×, with every ribbon drawn: by now every one of them belongs to that single strand.

- a spectre tile
- the previous page's block
- strands through the original tile
- the 9-tile supertile
- the original tile

The Spectre Knot

One tile, front and back

Front: the knot tile. Back: what it is.
Edge length 41 mm.



- 1 Cut round the outer black line, as one piece.
- 2 Fold along the dashed line, printed sides out.
- 3 Glue the halves back to back.

Make several and lay them edge to edge: the ribbons meet where the edges touch and cross there at right angles.